

Study and Comparison of MRI Image Denoising using Dual-Tree Complex DWT and Double-Density Dual-Tree Complex DWT

Manisha Rajoriya¹ and Munendra Singh²

¹Deptt. of E&C Engg. SET, Sharda University, Greater Noida

²Department of Physics SBSR, Sharda University, Greater Noida

E-mail: ¹manisha.rajoriya@sharda.ac.in, ²munendra.singh@sharda.ac.in

Abstract—DWT (Discrete Wavelet Transform) is used for image denoising which is very powerful tool. But it suffers from shift sensitivity, absence of phase information, and poor directionality. To remove out these limitations, many researchers developed extensions to the standard DWT such as WP (Wavelet Packet Transform), and SWT (Stationary Wavelet Transform). These extensions are highly redundant and computationally intensive. Complex Wavelet Transform (CWT) is also an impressive option, complex-valued extension to the standard DWT. There are various applications of Redundant CWT (RCWT) in an image processing such as Denoising, Motion estimation, Image fusion, Edge detection, and Texture analysis. In this work, the focused application is the image denoising using two innovative techniques and the images are considered which are corrupted by a random noise.

In this paper, first two sections explain about introduction to the topic and regarding wavelet transform domain. Third section gives an idea about basics concepts of the system. Forth section illustrates the proposed systems. Last section gives results and discussion. Here promising results are compared with DWT extensions namely, Dual-Tree Complex DWT (DTCWT) and Double-Density Dual-Tree Complex DWT (DDTCWT).

Keywords: CWT, DWT, Dual-Tree Complex DWT, Double-Density Dual-Tree Complex DWT.

1. INTRODUCTION

Image denoising is a technique which removes out noise which is added in the original image. Noise reduction is an important part of image processing systems. An image is always affected by noise. Image quality may get disturbed while capturing, processing and storing the image. Noise is nothing but the real world signals and which are not part of the original signal. In images, noise suppression is a particularly delicate task. In this task, noise reduction and the preservation of actual image features are the main focusing parts.

The wavelet transform provides a multi resolution representation using a set of analyzing functions that are dilations and translations of a few functions (wavelets). It overcomes some of the limitations of the Fourier transform

with its ability to represent a function simultaneously in the frequency and time domains using a single prototype function (or wavelet) and its scales and shifts [2].

The wavelet transform comes in several forms. The critically-sampled form of the wavelet transforms provides the most compact representation; however, it has several limitations.

It lacks the shift-invariance property, and in multiple dimensions it does a poor job of distinguishing orientations, which is important in image processing. For some applications, improvements can be obtained by using an expansive wavelet transform in place of a critically-sampled one [1]. A denoising method is used to improve the quality of image corrupted by a lot of noise due to the undesired conditions for image acquisition. The image quality is measured by the peak signal-to-noise ratio (PSNR) or signal-to noise ratio (SNR). Traditionally, this is achieved by linear processing such as Wiener filtering [3]. Recently introduced Dual-Tree Complex DWT and Double-Density Dual-Tree Complex DWT can give best results in image denoising applications.

2. WAVELET TRANSFORM DOMAIN

A Fourier Transform (FT) is only able to retrieve the global frequency content of a signal, the time information is lost.

A multi-resolution analysis becomes possible by using wavelet analysis. The Wavelet Transform (WT) retrieves frequency and time content of a signal. The basic types of wavelet transform are namely, i) Continuous Wavelet Transform (CoWT) ii) Discrete Wavelet Transform (DWT) iii) Complex Wavelet Transform (CWT). A multi-resolution analysis is not possible with Fourier Transform (FT) and Short Time Fourier Transform (STFT) and hence there is a restriction to apply these tools in image processing systems; particularly in image denoising applications. The multi-resolution analysis becomes possible by using wavelet analysis. A Continuous Wavelet Transform (CoWT) is calculated analogous to the Fourier

transform (FT), by the convolution between the signal and analysis function. The Discrete Wavelet Transform uses filter banks to perform the wavelet analysis.

2.1 Complex wavelet transform

This is a newly introduced technique of DWT. Orthogonal wavelet decompositions, based on separable, multirate filtering systems have been widely used in image and signal processing, largely for data compression. Kingsbury introduced a very elegant computational structure, the Dual-Tree complex wavelet transform [5], which displays near-shift invariant properties. Other constructions can be found such as in [6]. Kingsbury [3] pointed out the problems of Mallat-type algorithms. These algorithms have the lack of shift invariance.

Complex wavelets have not been used widely in image processing due to the difficulty in designing complex filters which satisfy a perfect reconstruction property. To overcome this, Kingsbury proposed a Dual-Tree implementation of the CWT (DT CWT) [7], which uses two trees of real filters to generate the real and imaginary parts of the wavelet coefficients separately. The DWT suffers from the following two problems.

1. Lack of shift invariance - this results from the down sampling operation at each level. When the input signal is shifted slightly, the amplitude of the wavelet coefficients varies so much.
2. Lack of directional selectivity - as the DWT filters are real and separable the DWT cannot distinguish between the opposing diagonal directions.

The first problem can be avoided if the filter outputs from each level are not down sampled but this increases the computational costs significantly and the resulting undecimated wavelet transform still cannot distinguish between opposing diagonals since the transform is still separable.

To distinguish opposing diagonals with separable filters the filter frequency responses are required to be asymmetric for positive and negative frequencies. A good way to achieve this is to use complex wavelet filters which can be made to suppress negative frequency components. The Complex DWT has improved shift-invariance and directional selectivity than the separable DWT [6]-[7].

3. BASIC CONCEPTS OF THE SYSTEM

A filter bank plays an important role in wavelet transform applications. It consists of two banks namely, analysis filter bank and synthesis filter bank. The one dimensional filter

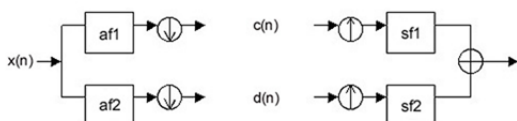


Fig. 1: One dimensional filter bank

bank is constructed with analysis and synthesis filter bank which is shown in Fig. 1.

The analysis filter bank decomposes the input signal $x(n)$ into two sub band signals, $c(n)$ and $d(n)$. The signal $c(n)$ represents the low frequency part of $x(n)$, while the signal $d(n)$ represents the high frequency part of $x(n)$. It uses filter banks to perform the wavelet analysis. The DWT decomposes the signal into wavelet coefficients from which the original signal can be reconstructed again. The wavelet coefficients represent the signal in various frequency bands. The coefficients can be processed in several ways, giving the DWT attractive properties over linear filtering.

3.1 A block schematic of wavelet based image denoising technique

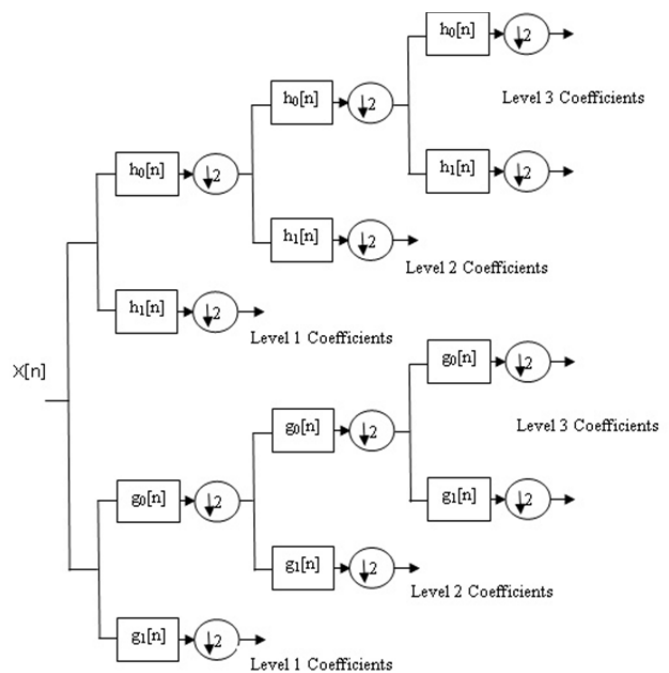


Fig. 2: Design implementation of Dual-Tree Complex DWT

Image denoising means usually compute the soft threshold in such a way that information present in image is preserved. A block schematic of Wavelet based image denoising technique is shown in Fig. 2.

Here the basic steps of wavelet based image denoising are given below.

- Decompose corrupted image by noise using wavelet transform.
- Compute threshold in wavelet domain and apply to noisy coefficients.
- Apply inverse wavelet transform to reconstruct image.

3.2 Basic differences between the two DWT extensions

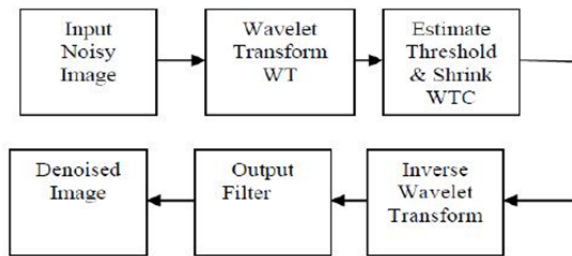


Fig. 3: A block schematic of wavelet based image denoising technique

The basic differences between the Dual-Tree DWT and Double-Density DWT are given below.

- The Dual-Tree and Double-Density DWTs are implemented with totally different filter bank structures.
- The Dual-Tree DWT can be interpreted as a complex-valued wavelet transform which is useful for signal modeling and denoising (the Double-Density DWT cannot be interpreted as such).
- For the Dual-Tree DWT there are fewer degrees of freedom for design, while for the Double-Density DWT there are more degrees of freedom for design.
- The Dual-Tree DWT can be used to implement two-dimensional transforms with directional wavelets, which is highly desirable for image processing [8].

4. PROPOSED SYSTEM

By introducing Complex wavelet transforms (CWT) concept, we can achieve Dual-Tree Complex DWT system. Also combining the Double-Density DWT and Dual-Tree Complex DWT, we can achieve the Double-Density Dual-Tree Complex DWT system. Complex wavelet transforms (CWT) use complex-valued filtering (analytic filter) that decomposes the real/complex signals into real and imaginary parts in transform domain. The real and imaginary coefficients are used to compute amplitude and phase information.

5. RESULTS AND DISCUSSION

The implementation of this work has performed in MATLAB software. Table 1 illustrates the result of MRI brain images for the RMS error with varying threshold (T) for DTDWT and Table 2 illustrates the results of DDDTDWT. Here as the noise increases DDDTCDWT gives better results than DTDWT.

Figure 8 and figure 12 are the original image corrupted by noise. Figure 9 and figure 10 are the output of denoised image by separable DWT. Figure 10 gives the output of denoised image by real DTDWT. Fig 11 gives output of denoised image by complex DTDWT.

Fig 14 shows the output of denoised image by real DDDTDWT. Fig 15 shows the output of denoised image by complex DDDTDWT.

6. CONCLUSION

The newly invented extensions of the DWT perform best in image processing applications. In this paper, the concept focused is wavelet based image denoising methods of an image which is corrupted by additive Gaussian noise. The techniques used are Dual-Tree Complex DWT and Double-Density Dual-Tree Complex DWT. These techniques give high performance as compared to the existing basic DWT methods. As noise increases Double-Density Dual-Tree Complex DWT works superior than Dual-Tree Complex DWT.

Table 1: Values of RMS error (with respect to original image) with varying T

Threshold value (T)	RMS error (decibal) w.r.t. original image			
	Noisy image	Separable DWT	Real 2D DTDWT	Complex 2D DTDWT
0	20.191	19.9503	20.0220	20.0345
5	20.191	16.4779	15.1624	14.4497
10	20.191	13.5972	11.3788	10.0345
15	20.191	11.3558	8.9617	7.6580
20	20.191	9.7770	7.8554	7.1757
25	20.191	8.8340	7.6604	7.5567
30	20.191	8.4248	7.9135	8.1244
35	20.191	8.3970	8.3272	8.6802
40	20.191	8.5934	8.3330	9.1939
45	20.191	8.9039	9.2161	9.6655
50	20.191	9.2579	9.6283	10.1043
55	20.191	9.6192	10.0174	10.5149
60	20.191	9.9719	10.3851	10.9007

Table 2: Values of RMS error (with respect to original image) with varying T

Threshold value (T)	RMS error (decibal) w.r.t. original image			
	Noisy image	Separable DWT	Real 2D DDDTDWT	Complex 2D DDDTDWT
0	20.191	19.9584	19.9584	19.9584
5	20.191	12.9315	11.9766	14.5116
10	20.191	9.2452	8.3965	10.5849
15	20.191	8.5529	8.5469	8.5691
20	20.191	9.1579	9.6790	8.1437
25	20.191	10.0760	10.8550	8.5398
30	20.191	11.0099	11.9270	9.1984
35	20.191	11.8905	12.8882	9.8878
40	20.191	12.7014	13.7510	10.5392
45	20.191	13.4436	14.5325	11.1385
50	20.191	14.1246	15.2240	11.6884
55	20.191	14.7509	15.8927	12.1949
60	20.191	15.3293	16.4870	12.6662

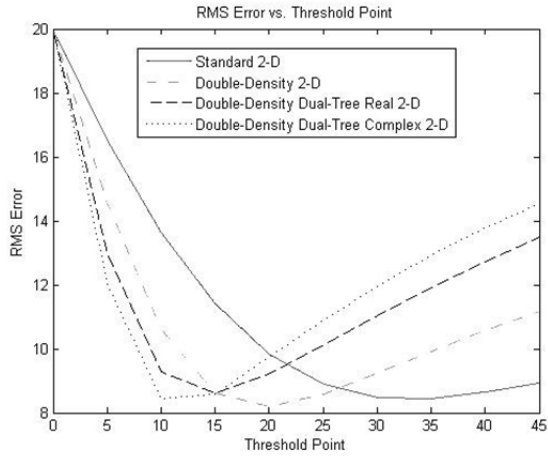


Fig. 4: Plot of RMS error versus threshold values for DDDTDWT

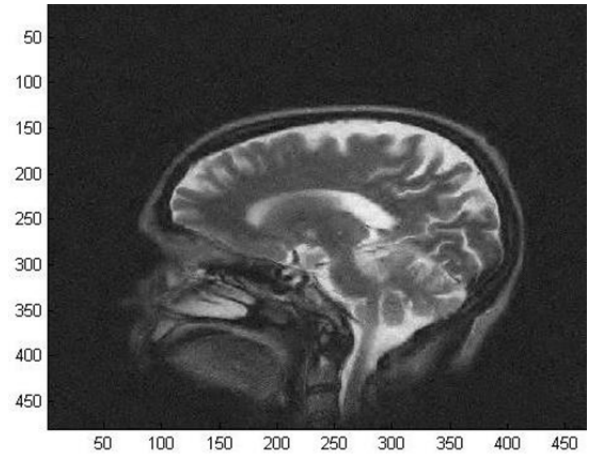


Fig. 10: Output of denoised image by separable DWT

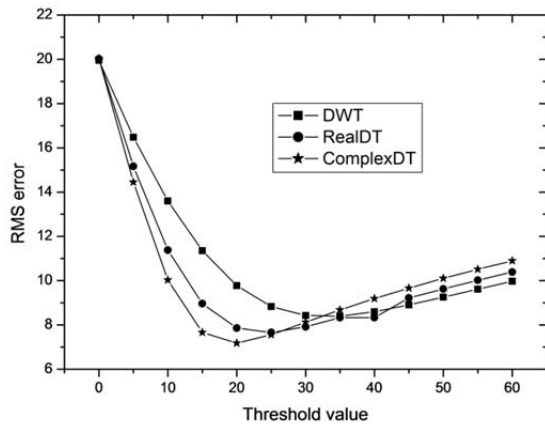


Fig. 5: Plot of RMS error versus threshold values for DTDWT

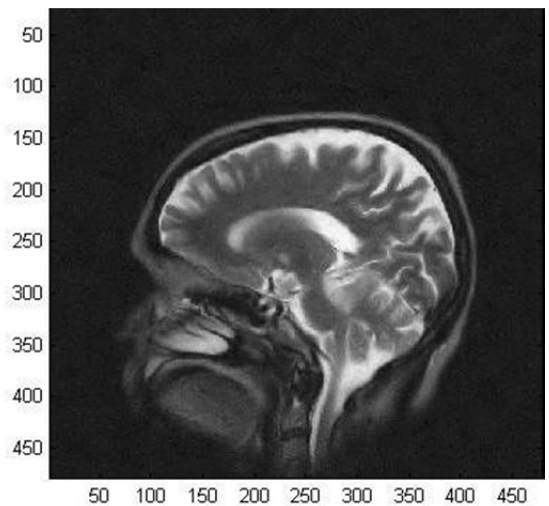


Fig. 11: Output of denoised image by real DTDWT

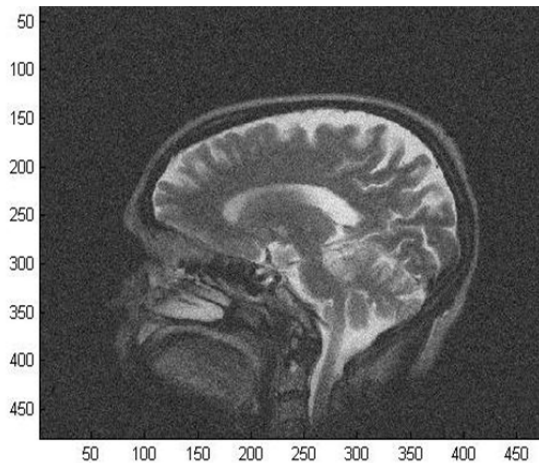


Fig. 6: Output of noisy image

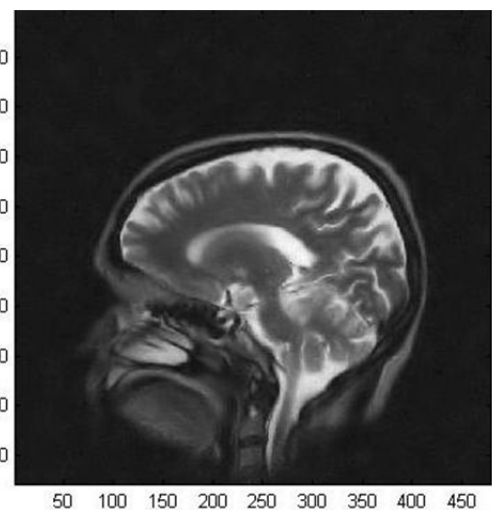


Fig. 12: Output of denoised image by real DTDWT

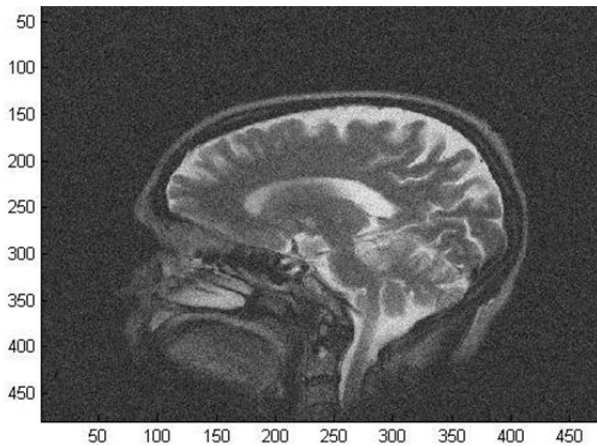


Fig. 13: Output of noisy image

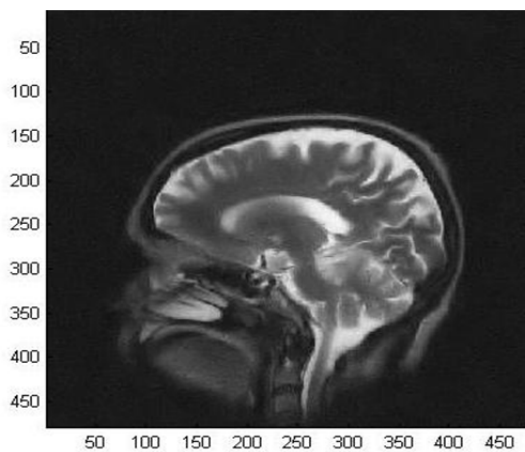


Fig. 14: output of denoised image by separable DWT

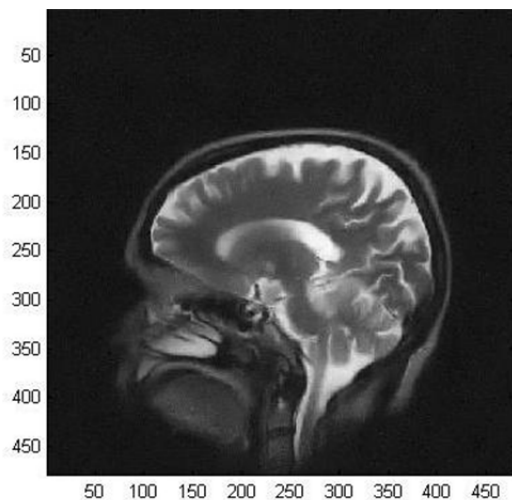


Fig. 15: Output of denoised image by real DDDTDWT

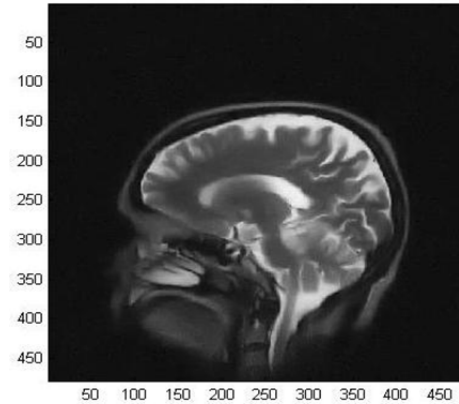


Fig. 16: Output of denoised image by complex DDDTDWT

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